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Large N spectrum of two matrices in a harmonic potential and BMN energies

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Abstract: The large N spectrum of the quantum mechanical hamiltonian of two hermitean matrices in a harmonic potential is studied in a framework where one of the matrices is treated exactly and the other is treated as a creation operator impurity in the background of the first matrix. For the free case, the complete set of invariant eigenstates and corresponding energies are obtained. When g^2_{YM} interactions are added, it is shown that a full string tension corrected BMN type hamiltonian is obtained.

Keywords: M(atrix) Theories, Penrose limit and pp-wave background, AdS-CFT Correspondence, 1/N Expansion.

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Contents

1. Introduction

Based on studies of giant gravitons in AdS x S and their SYM duals [1]–[8], a dual description of 1/2 BPS states has been arrived at in terms of free fermions associated with a complex matrix in a harmonic potential [8, 9]. More recently, it was established in [10] that the gravity description of 1/2 BPS states is completely determined by a density function associated with a general fermionic droplet configuration. Many other authors have studied this correspondence [11]–[34].

However, as emphasized recently in [35], the energy and flux of the 1/2 BPS states obtained in [10] are also those of a one dimensional hermitean matrix, in a bosonic phase space density description. In [35], an extension of 1/2 BPS states was studied by first treating this hermitean matrix exactly, and then by studying fluctuactions of states with "impurities" of the other matrix (in a creation/annihilation basis) about the large N background of the first matrix. A discrete linear spectrum of states was obtained, and a map to gravity states with either S or AdS radial dependence was proposed.

This work builds on earlier work [36, 37] where an approach to the derivation of string field theory from matrix model hamiltonians in the BMN limit was developed: in [36], the hamiltonian of free matrices in a harmonic potential resulting from compactification of scalars on $S^3 \times \mathbb{R}$ was considered, and the spectrum and cubic interactions of supergravity modes were obtained. The background was generated from known two point functions and the use of Schwinger-Dyson equations. In [37], a matrix model hamiltonian suitable to the pp wave limit in a creation/annhiliation basis was proposed that, once operator mixing is taken into account, was shown to correctly reproduce the pp wave light cone string field theory. Some knowledge of the properties of the underlying degrees of freedom is required.

The framework used in [35 − 37] is based on a change of variables from the original matrix degrees of freedom to gauge invariant collective matrix fields [38], whose spectra and interactions are susceptible of a gravity/string theory interpretation. This approach was used successfully in the developement of a string field theory of c=1 strings [39],[40].

In this article I investigate further the proposal of [35] that the large N properties of multi matrix models can be studied by solving one of the matrices exactly in the large N limit and by treating the other matrices in the backgound induced by that matrix in a systematic way.

A first check of this approach is to ensure that the full spectrum of gauge invariant states of two free hermitean matrices in a harmonic potential is recovered. This is carried out in this article, by generalizing the states considered in [35].

As pointed out in [35], one can think of the "impurity" as associated with the anti-holomophic complex matrix, but one can also associate it with one of the scalars in the transverse directions to the complex matrix plane. I elaborate on this last aspect of the problem. By identifying the analogue BMN [41] states and by carrying perturbation theory in g_{YM}, I obtain the first order string tension corrected BMN spectrum [41]. After performing a Bogoliubov transformation, I obtain a full string tension corrected BMN hamiltonian [42, 43].

This paper is organized as follows: in section 2, the large N limit of the quantum mechanics of two hermitean matrices in a harmonic potential is studied, with one of the matrices (the "second matrix") being treated in a creation/annihilation basis. The most general set of gauge invariant states is identified, and after treating the first matrix exactly, the leading and quadratic hamiltonian in terms of these gauge invariant states is arrived at [38, 44, 35]. The equation determining the spectrum of states with impurities is found to be a multi-local generalization of the Marchesini Onofri equation [45 − 49, 35], which appears in studies of the non-singlet sector of the single matrix theory. The spectrum is linear, with a large degeneracy. Chebyshev polynomials of the second kind play a special role in the description of the eigenfunctions.

In section 3, the result of introducing g_{YM} interactions is discussed. This first requires an identification of the original matrix variables used in section 2. The identification of matrix variables implied by the 1/2 BPS state description is reviewed, and I obtain the form of a typical F term in these variables. Writing down the analogue BMN [41] loops by quantum number matching, perturbation theory is carried out and the the first order g_{YM} corrected oscillator spectrum is obtained. I then discuss the case where the two matrices considered in section 2 are the real and imaginary part of a complex matrix, and the form of the coupling to a transverse hermitean matrix. By means of a Bogoliubov transformaton, the full string tension corrected expression for a BMN like hamiltonian is obtained.

I relate this result to the potential of an impurity in the presence of the single matrix backgound. Section 4 is reserved for a brief discussion and conclusions.

In the appendix, we describe how the local operators discussed in [35] are obtained from the general states introduced in this article by a suitable projection, and discuss some of their properties. .

2. Free case and multi local fields

2.1 Two matrix hamiltonian with a harmonic potential

We consider first the free case, i.e., the hamiltonian of two $N \times N$ hermitean matrices M and N in a harmonic potential. As is well known, the harmonic potential arises as a result of the curvature contribution in the leading Kaluza Klein compactification of the bosonic sector of $\mathcal{N} = 4$ SYM on $S^3 \times \mathbb{R}$. The exact physical identification of the matrices M and N will be discussed in the next section, although in general they will be linear combinations of two of the six adjoint scalar fields and their conjugate momenta.

The hamiltonian is

$$\hat{H} \equiv \frac{1}{2}\,\mathrm{Tr}(P_M^2) + \frac{w^2}{2}\,\mathrm{Tr}(M^2) + \frac{1}{2}\,\mathrm{Tr}(P_N^2) + \frac{w^2}{2}\,\mathrm{Tr}(N^2) \tag{2.1}$$

with P_M (P_N) canonical conjugate to M (N, respectively). As suggested in [35], we will from now on use a coherent state representaton for the matrix N:[1]

$$\hat{H} = \frac{1}{2}\,\mathrm{Tr}(P_M^2) + \frac{w^2}{2}\,\mathrm{Tr}(M^2) + w\,\mathrm{Tr}\left(B\frac{\partial}{\partial B}\right). \tag{2.2}$$

We are interested in the spectrum of gauge invariant states. One way to implement this invariance in the large N limit, is to restrict the hamiltonian to act on wave functionals of invariant single trace operators ("loops").

A complete set (in the large N limit) of gauge invariant operators is given by:

$$\begin{aligned}
\psi(k;0) &= \mathrm{Tr}(e^{ikM}) \\
\psi(k;1) &= \mathrm{Tr}(Be^{ikM}) \\
\psi(k_1,k_2;2) &= \mathrm{Tr}(Be^{ik_1M}Be^{ik_2M}) \\
&\cdots \\
\psi(k_1,k_2,\ldots,k_s;s) &= \mathrm{Tr}\left(\prod_{i=1}^{s}(Be^{ik_iM})\right), \qquad s>0\,.
\end{aligned} \tag{2.3}$$

Equivalently,

$$\begin{aligned}
\psi(x;0) &= \int \frac{dk}{2\pi}e^{-ikx}\psi(k;0) = \mathrm{Tr}(\delta(x-M)) \\
\psi(x;1) &= \int \frac{dk}{2\pi}e^{-ikx}\psi(k;1) = \mathrm{Tr}(B\delta(x-M)) \\
\psi(x_1,x_1;2) &= \int\!\!\int \frac{dk_1}{2\pi}\frac{dk_2}{2\pi}e^{-ik_1x_1}e^{-ik_2x_2}\psi(k_1,k_2;2)
\end{aligned}$$

[1] In this article, I will often switch from creation-annihilation operators to their coherent state representation $B^\dagger \to B$, $B \to \partial/\partial B$.

JHEP12(2005)043

$$
\begin{aligned}
&= \mathrm{Tr}(B\delta(x_1 - M)B\delta(x_2 - M)) \\
&\cdots \\
\psi(x_1, x_2, \ldots, x_s; s) &= \int \ldots \int \frac{dk_1}{2\pi} \ldots \frac{dk_s}{2\pi} e^{-ik_1x_1} \ldots e^{-ik_sx_s} \psi(k_1, k_2, \ldots, k_s; s) \\
&= \mathrm{Tr}\left(\prod_{i=1}^{s}(B\delta(x_i - M))\right), \qquad s > 0\,. \qquad (2.4)
\end{aligned}
$$

We will refer to these as "s impurity states" and, to simplify notation, we will often denote them by $\psi(A; s)$, with A an appropriate generic index.

The restriction of the action of the hamiltonian (2.2) on functionals of the invariant operators is implemented by performing a change of variables [38] from the original matrix variables to the invariant variables. Because of the reduction in the number of degrees of freeedom, the jacobian of this transformation has to be taken into account [38]. As loops with non-vanishing number of impurities have vanishing expectation values, this jacobian only depends on the zero impurity variables [44, 35]. Therefore, only the usual large N single matrix background of the matrix M is generated.

2.2 Background and zero impurity sector

The zero impurity sector is nothing but the large N hamiltonian dynamics of a single hermitean matrix. This is completely described by the standard cubic collective field hamiltonian (in addition to the original derivation [38] and its aplication to $c = 1$ strings [39, 40], reference [36] also has a general self-contained review of the method).

Including only the terms required for a study of the background and fluctuations, this hamiltonian takes the form:

$$
-\frac{1}{2}\int dx \partial_x \frac{\partial}{\partial\psi(x,0)}\psi(x,0)\partial_x \frac{\partial}{\partial\psi(x,0)} + \int dx \left(\frac{\pi^2}{6}\psi^3(x,0) + \psi(x,0)\left(\frac{w^2x^2}{2} - \mu\right)\right), \qquad (2.5)
$$

where the Lagrange multiplier μ enforces the contraint

$$
\int dx\psi(x,0) = N\,. \qquad (2.6)
$$

To exhibit explicitly the N dependence, we rescale

$$
\begin{aligned}
x &\to \sqrt{N}x \\
\psi(x,0) &\to \sqrt{N}\psi(x,0) \\
-i\frac{\partial}{\partial\psi(x,0)} \equiv \Pi(x) &\to \frac{1}{N}\Pi(x) \\
\mu &\to N\mu \qquad (2.7)
\end{aligned}
$$

and obtain

$$
\begin{aligned}
H^0_{\mathrm{eff}} &= \frac{1}{2N^2}\int dx \partial_x \Pi(x)\psi(x,0)\partial_x\Pi(x) + \\
&+ N^2\left(\int dx \frac{\pi^2}{6}\psi^3(x,0) + \psi(x,0)\left(\frac{w^2x^2}{2} - \mu\right)\right), \qquad (2.8)
\end{aligned}
$$

giving the well known Wigner distribution background in the limit as $N \to \infty$:

$$\pi\psi(x,0) \equiv \pi\phi_0 = \sqrt{2\mu - w^2x^2} = \sqrt{2w - w^2x^2}\,. \tag{2.9}$$

The droplet picture emerges naturally in this formalism: if we let [50] $p_\pm \equiv \partial_x\Pi(x)/N^2 \pm \pi\psi(x,0)$, then (2.8) has a very natural phase space representation as

$$H^0_{\rm eff} = \frac{N^2}{2\pi}\int_{p_-}^{p_+}\int dp dx\left(\frac{p^2}{2}+\frac{x^2}{2}-\mu\right).$$

As $N \to \infty$, the boundary of the droplet is given by $p_\pm^2 + x^2 = 2\mu$.

For the small fluctuation spectrum, one shifts

$$\psi(x,0) = \phi_0 + \frac{1}{\sqrt{\pi}N}\partial_x\eta\,; \qquad \partial_x\Pi(x) = -\sqrt{\pi}NP(x)$$

to find the quadratic operator

$$H^0_2 = \frac{1}{2}\int dx(\pi\phi_0)P^2(x) + \frac{1}{2}\int dx(\pi\phi_0)(\partial_x\eta)^2\,.$$

By changing to the classical "time of flight" ϕ

$$\frac{dx}{d\phi} = \pi\phi_0; \quad x(\phi) = -\sqrt{\frac{2}{w}}\cos(w\phi); \quad \pi\phi_0 = \sqrt{2w}\sin(w\phi); \quad 0 \le \phi \le \frac{\pi}{w}\,, \tag{2.10}$$

one obtains the hamiltonian of a free $1+1$ massless boson [39]:

$$H^0_2 = \frac{1}{2}\int d\phi P^2(\phi) + \frac{1}{2}\int d\phi(\partial_\phi\eta)^2\,. \tag{2.11}$$

Further imposition of Dirichelet boundary conditions at the classical turning points, for a consistent time evolution of the constraint (2.6), yields the spectrum in the zero impurity sector

$$\epsilon_j = wj\,; \qquad \phi_j = \sin(jw\phi)\,, \quad j = 1,2,\dots \tag{2.12}$$

The variable ϕ has a clear gravity interpretaton [35], as the angular variable in the plane of the droplet [10]. Finally, we note that the harmonic oscillator potential is special, in that the Wigner distribution background also safisfies the well known BIPZ [51] equation

$$\int dz\frac{\phi_0(z)}{(x-z)} = wx\,. \tag{2.13}$$

This result will turn out to be of importance in the following.

2.3 Spectrum of states with impurities — coarse graining

In [35] it was shown that the form of the quadratic operator determining the many-impurity spectrum is

$$H^s_2 = -\frac{1}{2}\sum_A \bar{\omega}(A,s)\frac{\partial}{\partial\psi(A,s)} + \frac{1}{2}\int dx\sum_A \Omega(x,0:A,s)\frac{\partial \ln J}{\partial\psi(x,0)}\frac{\partial}{\partial\psi(A,s)}\,, \tag{2.14}$$

where, to leading order in N

$$\partial_x \frac{\partial \ln J}{\partial \psi(x,0)} = \partial_x \int dy \Omega^{-1}(x,0;y,0)\omega(y,0) = 2\int dy \frac{\phi_0(y)}{(x-y)}\,. \tag{2.15}$$

In (2.14) and (2.15), $\omega(A,s)$ and $\Omega(x,0:A,s)$ have their usual meanings [38]:

$$\omega(A,s) = \mathrm{Tr}\left(\frac{\partial^2\psi(A,s)}{\partial M^2}\right) \tag{2.16}$$

$$\Omega(x,0:A,s) = \mathrm{Tr}\left(\frac{\partial\psi(0,x)}{\partial M}\frac{\partial\psi(A,s)}{\partial M}\right). \tag{2.17}$$

$\omega(A,s)$ splits the loop $\psi(A,s)$ and $\Omega(A,s:A',s')$ joins the two loops $\psi(A,s)$ and $\psi(A',s')$. $\bar{\omega}(A,s)$ indicates that only splittings into zero impurity loops need be considered. We have:

$$\Omega(k_0,0:k_1,\ldots,k_s,s) = -k_0\left(\sum_{i=1}^{s} k_i\psi(k_1,\ldots,k_i+k_0,\ldots,k_s;s)\right) \tag{2.18}$$

$$\bar{\omega}(\{k_i\}) = -2\sum_{i=1}^{s}\int_0^{k_i} dk'k'\psi(k_i-k';0)\psi(k_1,\ldots,k_{i-1},k',k_{i+1},\ldots,k_s;s)\,.$$

Equivalently

$$\Omega(z,0:x_1,\ldots,x_s,s) = \sum_{i=1}^{s}\frac{\partial}{\partial z}\frac{\partial}{\partial x_i}\left(\delta(z-x_i)\psi(x_1,\ldots,x_i,\ldots,x_s;s)\right) \tag{2.19}$$

$$\begin{aligned}\bar{\omega}(\{x_i\}) = &-2\sum_{i=1}^{s}\int dz\psi(z:0)\frac{\partial}{\partial x_i}\left(\frac{\psi(\{x_i\};s)}{x_i-z}\right) - \\ &-2\sum_{i=1}^{s}\int dz\psi(z;0)\times \\ &\times\left[\frac{\psi(\{x_i\};s)}{(x_i-z)^2} - \delta(z-x_i)\int dy_i\frac{\psi(x_1,\ldots,y_i,\ldots,x_s;s)}{(y_i-z)^2}\right].\end{aligned} \tag{2.20}$$

Substituting these in (2.14) and making use of (2.15), we obtain for the quadratic many-impurity operator

$$\begin{aligned}H_2^s = &\int dx_1\cdots\int dx_s\int dz\,\times \\ &\times\sum_{i=1}^{s}\frac{\phi_0(z)\psi(\{x_i\};s)-\phi_0(x_i)\psi(x_1,\ldots,z,\ldots,x_s;s)}{(x_i-z)^2}\frac{\partial}{\partial\psi(\{x_i\};s)}\,.\end{aligned} \tag{2.21}$$

The rescaling (2.7) implies that

$$\psi(A,s)\frac{\partial}{\partial\psi(A,s)} \to \frac{1}{N^{\frac{s}{2}}}\psi(A,s)\frac{\partial}{\partial\psi(A,s)}\,,$$

so that (2.14)(and (2.21)) is invariant, i.e., of order 1 (N^0) in N. Writing it as

$$\sum_A\sum_B\psi(A,s)K(A,B:s)\frac{\partial}{\partial\psi(B,s)}\,,$$

we obtain

$$\sum_B \psi(A,s)K(A,B;s)\frac{\partial}{\partial\psi(B,s)} = \tag{2.22}$$
$$= \sum_{i=1}^{s}\int dx_1 \ldots \int dx_s \psi(\{x_i\};s)\int dz \frac{\phi_0(z)}{(x_i-z)^2}\left(\frac{\partial}{\partial\psi(\{x_i\};s)} - \frac{\partial}{\partial\psi(x_1,\ldots,z,\ldots,x_s;s)}\right).$$

The s many-impurity kernel is then

$$K(\{x_i\},\{x_i'\};s) = \sum_{i=1}^{s}\int dz \frac{\phi_0(z)}{(x_i-z)^2}\left(\left(\prod_{j\neq i}\delta(x_j-x_j'))(\delta(x_i-x_i')-\delta(z-x_i')\right)\right). \tag{2.23}$$

When acting on an eigenfunctional

$$\Phi = \int \ldots \int dw_1 \ldots dw_s f(w_1,\ldots,w_s)\psi(w_1,\ldots,w_s:s),$$

we obtain

$$\sum_{i=1}^{s}\int dz \frac{\phi_0(z)}{(x_i-z)^2}(f(x_1,\ldots,x_s) - (f(x_1,\ldots,z,\ldots,x_s) = \tag{2.24}$$
$$= \sum_{i=1}^{s}\left(\left(-\frac{d}{dx_i}\int dz \frac{\phi_0(z)}{x_i-z}\right) f(x_1,\ldots,x_s) + \frac{d}{dx_i}\int dz \frac{\phi_0(z)f(x_1,\ldots,z,\ldots,x_s)}{x_i-z}\right)$$

i.e., a sum of Marchesini-Onofri kernels [45 − 47, 49, 35]. The eigenvalues and eigenfuntions of this operator follow from (2.13) and the result

$$\int_{-\sqrt{2/w}}^{\sqrt{2/w}} \frac{dz}{\pi}\frac{\sin(nw\phi(z))}{x-z} = -\cos(nw\phi(x)), \tag{2.25}$$

which, for the harmonic potential, is related to a well known integral relationship between the two different kinds of Chebyshev polynomails. The eigenvalues and eigenfuntions are

$$\epsilon_{n_i}' = w\left(\sum_{i=1}^{s} n_i - s\right); \qquad \Psi^s_{n_1,\ldots,n_s}(x_i) = \prod_{i=1}^{s}\frac{\sin(n_i w\phi(x_i))}{\sqrt{2w}\sin(w\phi(x_i))}; \quad n_i = 1,2,\ldots$$

We recognize the Chebyshev polynomials of the second kind $U_{n_i-1}(w\cos(\phi))$. Adding the contribution from the $\mathrm{Tr}(B\partial/\partial B)$ term of the hamiltonian we obtain

$$\begin{aligned}\epsilon_{n_i} = \sum_{i=1}^{s} n_i; \quad \Psi^s_{n_1,\ldots,n_s}(x_i) &= \prod_{i=1}^{s}\frac{\sin(n_i w\phi(x_i))}{\sqrt{2w}\sin(w\phi(x_i))}; \quad n_i = 1,2,\ldots \\ &= \prod_{i=1}^{s} u_{n_i-1}(x_i); \quad n_i = 1,2,\ldots,\end{aligned} \tag{2.26}$$

where to simplify notation I introduced the polynomials

$$u_{n-1}(x) \equiv \frac{\sin(nw\phi(x))}{\sqrt{2w}\sin(w\phi(x))} = \frac{\sin(nw\phi(x))}{\pi\phi_0(x)} = \frac{1}{\sqrt{2w}}U_{n-1}\left(-\sqrt{\frac{w}{2}}x\right). \tag{2.27}$$

We see a one to one correspondence with the spectrum of the states $\mathrm{Tr}(A^{\dagger m_1}B^{\dagger l_1}A^{\dagger m_2}B^{\dagger l_2}\ldots)$, obtained by acting on the original Fock space of the theory.

2.4 Measure and Fock space

The states obtained in the previous subsection:

$$\langle\psi|\{n_i\},s\rangle = \int_{-\sqrt{2/w}}^{\sqrt{2/w}} dx_1 \dots \int_{-\sqrt{2/w}}^{\sqrt{2/w}} dx_s \{u_{n_1-1}(x_1)\dots u_{n_s-1}(x_s)\}_c \psi(\{x_i\};s) \qquad (2.28)$$

($\{\dots\}_c$ stands for cyclic symmetrization) form a complete orthonormal set with respect to the measure

$$\int [d\psi] \exp\left[-\int \frac{dx_1}{\pi\phi_0}\dots\int \frac{dx_s}{\pi\phi_0}|\psi(\{x_i\};s)|^2\right]. \qquad (2.29)$$

This is because the polynomials $u_n(x)$ form an orthonormal set with weight function $\pi\phi_0 = \sqrt{2w - w^2x^2}$. In terms of the original matrix variables, these states take the form

$$\langle\psi|\{n_i\};s\rangle = \mathrm{Tr}\left(Bu_{n_1-1}(M)Bu_{n_2-1}(M)\dots Bu_{n_s-1}(M)\right). \qquad (2.30)$$

These states, with their well defined energies, provide the analogue of the usual states with well defined U(1) charges built as traced products of complex monomials. They also have a more natural ϕ space representation, i.e with

$$\psi(\phi_1,\dots,\phi_s;s) = \psi(x_1(\phi_1),\dots,x_s(\phi_s);s),$$

then

$$\langle\psi|\{n_i\};s\rangle = \int_0^{\pi/w} d\phi_1 \dots \int_0^{\frac{\pi}{w}} d\phi_s \sin(n_1 w\phi_1)\dots\sin(n_s w\phi_s)\psi(\{\phi_i\},s) \qquad (2.31)$$

and the measure (2.29) is trivial:

$$\int [d\psi] \exp\left[-\int d\phi_1 \dots \int d\phi_s |\psi(\{\phi_i\},s)|^2\right].$$

The orthonormality of the polynomials $u_n(x)$ is then seen to be a simple consequence of the orthogonality of the sin functions. Equation (2.31) also exhibits a one to one correspondence between the eigenfunctions of multi-impurity states, and those of the zero-impurity sector, which are solutions of a $1+1$ dimensional massles Klein-Gordon equation. This is not entirely surprising, as it is known that the Marchesini-Onofri operator squares to the massless Klein-Gordon equation [35] (and earlier references therein). Indeed, the expectation is that in a full treatment where the second matrix is kept hermitean, all quadratic fluctuations will be determined by massless Klein-Gordon equations in the ϕ variable. A very natural string Fock space suggests itself for the second quantized fields $\psi(\{x_i\};s)$ which can be constructed along the lines of [36, 37]. This will not be pursued in this article.

3. g_{YM} interactions pp- Stringy hyperfine splitting and BMN spectrum

3.1 1/2 BPS variables

We briefly review the precise identification of variables resulting from the dual description of 1/2 BPS states [8]. Starting with the leading Kaluza Klein compactification of the bosonic sector of $\mathcal{N}=4$ SYM on $S^3\times\mathbb{R}$, and choosing the plane defined by two scalars X_1 and X_2 grouped into a complex matrix $Z=X_1+iX_2$, we introduce matrix valued creation and annhilation operators[2]

$$Z=\frac{1}{\sqrt{w}}(A+B^\dagger) \qquad \Pi=-i\frac{\sqrt{w}}{2}(A^\dagger-B)\,.$$

Π is the canonical conjugate to Z. Motion in this plane is characterized by the energy and the two dimensional angular momentum of the free theory:

$$\hat{H}_0=w(\,\mathrm{Tr}(A^\dagger A)+\mathrm{Tr}(B^\dagger B)) \qquad \hat{J}=\mathrm{Tr}(A^\dagger A)-\mathrm{Tr}(B^\dagger B)\,. \tag{3.1}$$

B carries a well defined quantum of charge -1, and 1/2 BPS states correspond to a restriction to the sector with no B excitations. Therefore B is the impurity considered in [35]. M is the hermitean matrix associated with the A, $A^\dagger$ system [35]:

$$M\equiv\frac{1}{\sqrt{2w}}(A+A^\dagger) \quad P_M=-i\sqrt{\frac{w}{2}}(A+A^\dagger)\,. \tag{3.2}$$

We can also project out the B sector by taking a pp wave limit [41], while retaining one the harmonic modes, denoted by C, associated with one of the (complex) transverse directions [37]. The interaction hamiltonian splits into F and D terms. There are well known non-renormalization theorems that apply to the D term contributions [52], so we will concentrate on the F term interaction hamiltonian[3]

$$H_{\rm int}=-\frac{g_{YM}^2}{w^2}\,\mathrm{Tr}([A^\dagger,C^\dagger][A,C])\,. \tag{3.3}$$

This term was shown in [37] to generate fully the scalar sector of the pp wave light cone string field theory [53], and it agrees with the leading order contribution of other hamiltonian based, but different, approaches [54, 55]. The C oscillator does not participate in the Z plane $\hat{J}$ charge, but it contributes to the energy, so free states with s C impurities only can be classified as eigenstates of the operators

$$\begin{aligned}
\hat{J}\;&=\mathrm{Tr}(A^\dagger A)=\frac{1}{2}\,\mathrm{Tr}(P_M^2)+\frac{w^2}{2}\,\mathrm{Tr}(M^2) \qquad && j=\sum_{i=1}^{s}n_i-s\\
\hat{H}_0\;&=\frac{1}{2}\,\mathrm{Tr}(P_M^2)+\frac{w^2}{2}\,\mathrm{Tr}(M^2)+w\,\mathrm{Tr}(C^\dagger C) \qquad && \epsilon_j=\sum_{i=1}^{s}n_i=j+s\,,
\end{aligned} \tag{3.4}$$

where we have used the results of Section 2. For instance, specializing to two impurities, all states with $j=j_1+j_2, j_1\geq 0, j_2\geq 0$ $(j_i=n_i-1)$ are degenerate. The analogue BMN [41] operators are now easily contructed

$$\begin{aligned}
O_J^m\;&=\frac{1}{\sqrt{J}}\sum_{j=0}^{J}e^{\frac{2\pi i m}{J+1}j}\langle\psi|j,J-j;2\rangle\\
&=\frac{1}{\sqrt{J}}\sum_{j=0}^{J}e^{\frac{2\pi i m}{J+1}j}\,\mathrm{Tr}(Cu_j(M)Cu_{J-j}(M))\,.
\end{aligned} \tag{3.5}$$

[2] w is inversely proportional to the S^3 radius, and it can be scaled out of the action. It may be set to 1 at the end of the calculation

[3] Throughout this article we consider all terms of the hamiltonian to be normal ordered.

We now wish to calculate the first order perturbation theory correction to the free energy $J+2$ of the BMN operator above. For this, we write (3.3) in terms of our hermitean variables (3.2) after noticing that the rescaling (2.7) requires that we let $M \to \sqrt{N}M$. The interaction hamiltonian then takes the form:

$$\begin{aligned}\hat{H}_{\rm int} &= -\frac{g_{YM}^2 N}{2w}\,\mathrm{Tr}([M,C][M,C^\dagger]) + i\frac{g_{YM}^2}{2w^2}\,\mathrm{Tr}([C,C^\dagger][M,P_M]) - \\ &\quad -\frac{g_{YM}^2}{2Nw^3}\,\mathrm{Tr}([P_M,C][P_M,C^\dagger]) \equiv \hat{H}_1 + \hat{H}_2 + \hat{H}_3\,. \qquad (3.6)\end{aligned}$$

We obtain

$$\hat{H}_1\,\mathrm{Tr}(Cu_j(M)Cu_{J-j}(M)) = \qquad (3.7)$$

$$\begin{aligned}&= \frac{g_{YM}^2 N}{w}\{-2\,\mathrm{Tr}(CMu_j(M)CMu_{J-j}(M)) + \\ &\qquad + \mathrm{Tr}(CM^2u_j(M)Cu_{J-j}(M)) + \mathrm{Tr}(Cu_j(M)CM^2u_{J-j}(M))\}\,. \qquad (3.8)\end{aligned}$$

Using the identity

$$U_{j+1}(x) - 2xU_j(x) + U_{j-1}(x) = 0\,,$$

and keeping terms with $j_1+j_2 = J$ only, as we are only interested in the first order correction to the energy, and states with different J are orthogonal as explained in subsection 2.4, we obtain:

$$\begin{aligned}\hat{H}_1\langle\psi|j,J-j;2\rangle &= \frac{g_{YM}^2 N}{w^2}(2\langle\psi|j,J-j;2\rangle - \\ &\qquad - \langle\psi|j+1,J-j-1;2\rangle - \langle\psi|j-1,J-j+1;2\rangle)\,, \qquad (3.9)\end{aligned}$$

it follows from (3.5) that the first order shift in the BMN loop energy coming from $\hat{H}_1$ is

$$\Delta\epsilon(O_J^m) = \frac{g_{YM}^2 N}{J^2}\left(\frac{2\pi m}{w}\right)^2\,, \qquad (3.10)$$

where as usual $\lambda' = g_{YM}^2 N/J^2$ is kept finite as $J\to\infty$ Typically, $\hat{H}_2$ and $\hat{H}_3$ will split the loop $\langle\psi|j,J-j;2\rangle$ into states with $0+2$, $1+1$, $0+0+2$ and $0+1+1$ impurities. Because 0 impurity states develop a background, $0+2$ and $0+0+2$ states can potentially correct the energy. However, we are only interested in establishing if they generate a correction to the result (3.10). From the N dependence of $\hat{H}_2$ and $\hat{H}_3$ in (3.6), it is clear that any other possible dependence on the 't Hooft coupling comes from the terms with $\mathrm{Tr}\,1 = N$. We have:

$$\begin{aligned}&\hat{H}_2\,\mathrm{Tr}(Cu_j(M)Cu_{J-j}(M)) = \\ &= \frac{g_{YM}^2}{w^2}\{2N\,\mathrm{Tr}(Cu_j(M)Cu_{J-j}(M)) - \mathrm{Tr}(u_j)\,\mathrm{Tr}(C^2u_{J-j}(M)) - \mathrm{Tr}(u_{J-j})\,\mathrm{Tr}(C^2u_j)\} + \\ &\quad + \text{interactions}\,. \qquad (3.11)\end{aligned}$$

The terms proportional to the 't Hooft coupling are clearly not universral, as the only other finite parameter is $g_2 = J^2/N$. For $\hat{H}_3$,we have

$$\hat{H}_3 \,\mathrm{Tr}(Cu_j(M)Cu_{J-j}(M)) = \frac{g_{YM}^2 N}{w^3}\Big\{ 2\,\mathrm{Tr}\left(Cu_j^{(1)}(M)Cu_{J-j}^{(1)}(M)\right) - \\ - \mathrm{Tr}\left(Cu_j^{(2)}(M)Cu_{J-j}(M)\right) - \\ - \mathrm{Tr}\left(Cu_j(M)Cu_{J-j}^{(2)}(M)\right)\Big\} + \cdots ,$$

where I have defined

$$u_j^{(1)}(x) = \frac{u_j - a_j^0}{x} \qquad u_j^{(2)}(x) = \frac{u_j - a_j^1 x - a_j^0}{x^2} \qquad u_j(x) = a_j^0 + a_j^1 x + \cdots + a_j^j x^j \,. \quad (3.12)$$

Clearly, these polynomials can be written as linear combinations of Chebyshev polynomials of strictly lower order then j, so again the above terms do not contribute, and the result (3.10) is not corrected to this order. As is well well known, this is the expected result.

It would be very interesting to develop the form of the string field theory emerging from this matrix model along the lines of [37]. This is beyond the scope of this paper.

3.2 Real-imaginary variables — BMN spectrum

We now consider the identification of the matrices M and N as the real and imaginary part of a single complex matrix Z. Again, we will retain a typical term arising from the coupling to one of transverse hermitean scalars denoted by Q:

$$\begin{aligned} \hat{H}_{\mathrm{int}} &= -g_{YM}^2 N\,\mathrm{Tr}([M,Q]^2) \\ &= -\frac{g_{YM}^2 N}{2w}\left(\mathrm{Tr}([M,C]^2 + 2[M,C^\dagger][M,C] + [M,C^\dagger]^2\right). \end{aligned} \quad (3.13)$$

We recognize the impurity number conserving term, up to a factor, as $\hat{H}_1$ in (3.6). As explained in the previous subsection, its action on a BMN loop (3.5) leads to a first order shift in energy:

$$\Delta\epsilon(O_J^m) = 2\frac{g_{YM}^2 N}{J^2}\left(\frac{2\pi m}{w}\right)^2 \quad (3.14)$$

However, the form of the interaction in (3.13) suggests that it may be possible to eliminate the CC and $C^\dagger C^\dagger$ terms by means of a Bogoliubov transformation

$$C_{ij} = \cosh(\phi_{ij})\tilde{C}_{ij} - \sinh(\phi_{ij})\tilde{C}_{ij}^\dagger$$

This is indeed the case provided

$$\tanh(2\phi_{ij}) = \frac{\frac{g_{YM}^2 N}{w}(\lambda_i - \lambda_j)^2}{w + \frac{g_{YM}^2 N}{w}(\lambda_i - \lambda_j)^2} \,,$$

where the λ_i's are the eigenvalues of the matrix M. Then

$$\hat{H} = \cdots + \sum_{i,j=1}^{N} \sqrt{\left(w + \frac{g_{YM}^2 N}{w}(\lambda_i - \lambda_j)^2\right)^2 - \left(\frac{g_{YM}^2 N}{w}(\lambda_i - \lambda_j)^2\right)^2} \quad \bar{C}^\dagger_{ij}\bar{C}_{ji}$$
$$= \cdots + \sqrt{w^2 + 2g_{YM}^2 N(\lambda_i - \lambda_j)^2} \quad \bar{C}^\dagger_{ij}\bar{C}_{ji}\,. \tag{3.15}$$

In the above, $\bar{C} = V^\dagger \tilde{C} V$, where V is the unitary matrix that diagonalizes M and ... denotes normal ordering contributions. But

$$\sum_{i,j=1}^{N} \bar{C}^\dagger_{ij}\sqrt{w^2 + 2g_{YM}^2 N(\lambda_i - \lambda_j)^2}\bar{C}_{ij} = \tag{3.16}$$
$$= w\,\mathrm{Tr}(\tilde{C}^\dagger\tilde{C}) - \frac{g_{YM}^2 N}{w}\,\mathrm{Tr}([M,\tilde{C}^\dagger][M,\tilde{C}]) - \frac{(g_{YM}^2 N)^2}{2w^3}\,\mathrm{Tr}([M,[M,\tilde{C}^\dagger]][M,[M,\tilde{C}]]) + \cdots\,.$$

The term linear in $g_{YM}^2 N$ is the impurity conserving term in (3.13). The above expansion completes the square root [56, 41].

The above result can be understood as follows: the dynamics of the transverse hermitean scalar Q in the background of the scalar M is determined by the potential

$$V_Q = \frac{w^2}{2}\,\mathrm{Tr}(Q^2) - g_{YM}^2 N\,\mathrm{Tr}([M,Q]^2)\,.$$

The large N dynamics of M is the large N dynamics of its eigenvalues described by the density of eigenvalues (2.4), (2.7)

$$\frac{1}{N}\sum_{i=1}^{N}\delta(x - \lambda_i) \equiv \psi(x,0) \to \phi_0 \quad \text{as} \quad N \to \infty\,.$$

Rewritting V_Q, as suggested in [43], in the form

$$V_Q = \sum_{i,j=1}^{N} \frac{w^2}{2}\bar{Q}_{ij}\bar{Q}_{ji} + g_{YM}^2 N \bar{Q}_{ij}(\lambda_i - \lambda_j)^2\bar{Q}_{ji}\,,$$

shows that each of the cordinates $\bar{Q}_{ij}$ has a (background dependent) frequency $\bar{w}_{ij}$ given by

$$\bar{w}_{ij}^2 = w^2 + 2g_{YM}^2 N(\lambda_i - \lambda_j)^2\,, \tag{3.17}$$

in complete agreement with (3.15).

This calculation is similar in spirit to that of [43] in the sense that there too the spectrum was calculated about a large N background, but of commuting matrices. We also expand about a background, that of a single matrix, but there is no need to assume commutativity.

4. Conclusion

We have sucessfully applied the idea that the large N limit of a system of matrices with a harmonic potential can be studied by treating one of them exactly and the others, in a creation/annihilation basis, in the background of the first, to two hermitean in the free case and then to BMN loops. The results of this aricle generalize in a straightforward way to the case of impurities of different types. This will be reported in a forthcoming publication [57].

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A. Free case — local states

In [35], (super) gravity states $\psi_S(k,s)$ (or $\psi_S(x,s)$) were introduced:

$$\begin{aligned}
\psi_S(k,1) &= \mathrm{Tr}(Be^{ikM}) = \psi(k,1) \\
\psi_S(k,2) &= \int_0^k dk_1\, \mathrm{Tr}(Be^{ik_1M}Be^{i(k-k_1)M}) = \int_0^k dk_1 \psi(k_1, k-k_1) \\
\psi_S(k,3) &= \int_0^k dk_2 \int_0^{k_2} dk_1\, \mathrm{Tr}(Be^{ik_1M}Be^{i(k_2-k_1)M}Be^{i(k-k_2)M}) \\
&= \int_0^k dk_2 \int_0^{k_2} dk_1 \psi(k_1, k_2-k_1, k-k_2; 3) \\
&\cdots
\end{aligned} \tag{A.1}$$

with $\psi_S(x,s)$ obtained by Fourier transforms. The spectrum of these states was found in [35], where it was also found to be linear. Here we explain how the same spectrum can be obtained from the more general states above, by a suitable projection. As an example, let us consider two impurities. One can easily establish that

$$\begin{aligned}
\psi_S(k:2) &= -i \int dx_1 \int dx_2 \psi(x_1,x_2:2) \int dk' e^{ik'x_1} e^{i(k-k')x_2} \\
&= -2i \int dx_1 \int dx_2 \psi(x_1,x_2:2) \frac{e^{ikx_1}}{x_1-x_2}\,,
\end{aligned} \tag{A.2}$$

and hence

$$\psi_S(x:2) = -2i \int dy \frac{\psi(x,y:2)}{x-y}\,.$$

We can now let the kernel (2.23) act on states

$$\int dx_1 g(x_1) \psi_S(x_1:2) = -i \int dx_1 \int dx_2 \left(\frac{g(x_1)}{x_1-x_2} - \frac{g(x_2)}{x_1-x_2} \right).$$

So instead of generic functions $f(x_1, x_2)$, the kernel acts on functions of the form

$$f(x_1, x_2) = \frac{g(x_1)}{x_1 - x_2} - \frac{g(x_2)}{x_1 - x_2}.$$

One obtains ($w = 1$ in this discussion)

$$\begin{aligned} \hat{K}\left[\frac{g(x_1)}{x_1 - x_2}\right] &= \frac{g(x_1)}{x_1 - x_2}\left(\int dz \frac{\phi_0(z)}{(x_1 - z)^2} + \int dz \frac{\phi_0(z)}{(x_2 - z)^2}\right) - \\ &\quad - g(x_1)\int dz \frac{\phi_0(z)}{(x_2 - z)^2}\frac{1}{x_1 - z} - \int dz \frac{\phi_0(z)}{(x_1 - z)^2}\frac{g(z)}{z - x_2}. \end{aligned} \tag{A.3}$$

By setting

$$g(x) = \Phi_n(x) = \frac{\sin(n\phi(x))}{\sqrt{2}\sin(\phi(x))},$$

use of (2.13), (2.25) or their derivatives together with the use of partial fractions yields

$$\hat{K}\left[\frac{g(x_1)}{x_1 - x_2}\right] = -2\frac{g(x_1)}{x_1 - x_2} + \frac{\cos(n\phi(x_1)) - \cos(n\phi(x_2))}{(x_2 - x_1)^2} + n\frac{g(x_1)}{x_2 - x_1}.$$

Adding the contribution obtained by the cyclic permutation $1 \to 2, 2 \to 1$, we obain:

$$\hat{K}\left[\frac{g(x_1) - g(x_2)}{x_1 - x_2}\right] = (n-2)\left(\frac{g(x_1) - g(x_2)}{x_1 - x_2}\right).$$

This pattern generalizes. For instance, for three impurities, one takes

$$f(x_1, x_2, x_3) = \frac{g(x_3)}{(x_3 - x_1)(x_3 - x_2)} + \frac{g(x_1)}{(x_1 - x_2)(x_1 - x_3)} + \frac{g(x_2)}{(x_2 - x_3)(x_2 - x_1)}$$

and the eigenvalue is $n - 3$. For s impurities, and taking into account the $\mathrm{Tr}(B\partial/\partial B)$ term of the hamiltonian we obtain

$$w_n = n; \qquad \Phi_n^s(z) = \frac{\sin(nq(z))}{\sqrt{2}\sin(q(z))}; \qquad n = 1, 2, \ldots$$

This confirms the results of [35]. It is important to realize that these states are normalized with respect to measure different from the multi-local states discussed previously. For instance, for two impurities, the measure is

$$\int [d\psi_S] \exp\left[-\int \frac{dx}{\pi\phi_0}|\psi_S(x;2)|^2\right] \tag{A.4}$$

but

$$\int \frac{dx}{\pi\phi_0}|\psi_S(x;2)|^2 = 4\int \frac{dx}{\pi\phi_0}\int dx_1 \int dx_2 \frac{1}{x - x_1}\frac{1}{x - x_2}\psi^*(x, x_1)\psi(x, x_2)$$

which is clearly distinct from (2.29), explaining the existence of another sequence of states.

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